Transactions of Tianjin University
ISSN 1006-4982 pp376-380
Vol.11 No. 5 Oct. 2005

Earthquake Analysis for the System of RC Building with a Steel Tower

WANG Yiqun(王依群)[1], TENG Xiangquan(滕祥泉)[2], AN Guoting(安国亭)[1]
(1. School of Civil Engineering, Tianjin University, Tianjin 300072, China;
2. Electronic Engineering Design Institute of China, Beijing 100840, China)

Abstract:A steel tower topping an RC building comprises a non-proportional damping structural system with different damping ratios. To compare the results from the non-proportional damping model and the equivalent damping model.the structural system was calculated with the two damping models during earthquake respectively, using earthquake time history analysis computer program developed by the authors. Differences in the calculated results of inner forces and displacements using the two damping models were observed. It is found that if the equivalent damping model is used in design, the consequence will be unsafe for the steel tower and too safe for the RC building at the same time.
Keywords:non-proportional damping; equivalent damping ratio; dynamic time history analysis

The damping ratios of RC buildings and steel tower are 0. 05 and 0. 01 respectively, so a steel tower topping an RC building comprises a non-proportional damping system[1,2]. Due to the difficulty of dynamic analysis for non-proportional damping systems[3-5], these systems are usually simplified as proportional damping ones with equivalent damping ratio. The equivalent damping ratio takes a value more than 0.01 and less than 0. 05.

To compare the results calculated using the non-proportional damping model and the equivalent damping model is the main aim of this paper. First, a direct dynamic analysis method for the systems with non-proportional damping is presented. Second, an example of an RC building with a steel tower atop it during earthquake is calculated by using the method with non-proportional damping model and the equivalent proportional damping model. Then, the calculated results of the two models are compared and some differences are found.

1 Damping matrix of non-proportional damping system

Suppose the system is made up of n substructures. The damping property is in accordance with Rayleigh's supposition for each of the substructures. That is

$$[C_i]=\alpha_i[M_i]+\beta_i[K_i]\quad(1\leqslant i\leqslant n)\qquad(1)$$

where $[C_i]$, $[M_i]$ and $[K_i]$ are the damping, mass and stiffness matrixes of the ith substructure respectively ;α_i and β_i are the Rayleigh's coefficients of the ith substructure.

The mass, stiffness and damping matrixes of the global structure are

$$[M]=\sum_{i=1}^{n}[M_i]\qquad(2)$$

$$[K]=\sum_{i=1}^{n}[K_i]\qquad(3)$$

$$[C]=\sum_{i=1}^{n}[C_i]=\sum_{i=1}^{n}\alpha_i[M_i]+\sum_{i=1}^{n}\beta_i[K_i]\qquad(4)$$

Accepted date:2005- 05- 13.
WANG Yiqun, born in 1953,male,Dr,associate Prof.
E-mail: yqwangtj@ hotmail. com.

The formulas for the structures with materials of two different damping properties, that is $n = 2$, are given below in brevity. Let the first kind of material be the one occupying the major part of the structure, the other one be the second. So the damping matrix of the whole structure can be seen as the one made up of the first kind of material, but some changes must be made for the part of structure made up of the second kind of material.

$$[C] = \alpha_1[M_1] + \beta_1[K_1] + \alpha_2[M_2] + \beta_2[K_2] = \alpha_1([M_1] + [M_2]) + \beta_1([K_1] + [K_2]) + (\alpha_2 - \alpha_1)[M_2] + (\beta_2 - \beta_1)[K_2]$$

Let $\alpha = \alpha_1$, $\beta = \beta_1$, $\Delta\alpha = \alpha_2 - \alpha$, $\Delta\beta = \beta_2 - \beta$, we get

$$[C] = \alpha[M] + \beta[K] + \Delta\alpha[M_2] + \Delta\beta[K_2] \tag{5}$$

2 Direct integral algorithm

Some modal superposition methods[3,4] have been put foward for the systems with nonproportional damping. The accuracy of the results using these methods depends on the number of the modals used for superposition and on the number of the iterations. In this paper the direct integral method[6] is used.

The equation of dynamic equilibrium of the constant acceleration direct integration method is

$$\left[\frac{4}{\Delta t^2}[M] + \frac{2}{\Delta t}[C] + [K]\right]\{\Delta r\} = \{\Delta P\} + [M](2\{\ddot{r}_0\} + \frac{4}{\Delta t}\{\dot{r}_0\}) + [C]\{2\dot{r}_0\} \tag{6}$$

where $\{\Delta r\}$ and $\{\Delta P\}$ are the increments of displacement and load vectors respectively; $\{\dot{r}_0\}$ and $\{\ddot{r}_0\}$ are the velocity and acceleration vectors respectively.

Substituting Eq. (5) into Eq. (6) gives

$$\left[(\frac{4}{\Delta t^2} + \frac{2\alpha}{\Delta t})[M] + (\frac{2\beta}{\Delta t} + 1)[K] + \frac{2\Delta\alpha}{\Delta t}[M_2] + \frac{2\Delta\beta}{\Delta t}[K_2]\right]\{\Delta r\} = \{\Delta P\} + 2[M]\cdot\left[\{\ddot{r}_0\} + (\frac{2}{\Delta t} + \alpha)\{\dot{r}_0\}\right] + 2\beta[K]\{\dot{r}_0\} + 2\Delta\alpha[M_2]\{\dot{r}_0\} + 2\Delta\beta[K_2]\{\dot{r}_0\} \tag{7}$$

To avoid the need to evaluate the term $2\beta[K]\{\dot{r}_0\}$, the transformation[7]

$$\{\Delta x\} = \{\Delta r\} + \beta\{\Delta\dot{r}\} = (\frac{2\beta}{\Delta t} + 1)\{\Delta r\} - 2\beta\{\dot{r}_0\} \tag{8}$$

is used. Eq. (7) can therefore be written as

$$\left[\gamma[M] + [K] + \frac{2}{2\beta + \Delta t}(\Delta\alpha[M_2] + \Delta\beta[K_2])\right]\{\Delta x\} = \{\Delta P\} + 2[M]\cdot\left[\{\ddot{r}_0\} + (\frac{2}{\Delta t} + \alpha - \beta\gamma)\{\dot{r}_0\}\right] + \frac{2\Delta t}{2\beta + \Delta t}(\Delta\alpha[M_2] + \Delta\beta[K_2])\{\dot{r}_0\} \tag{9}$$

where

$$\gamma = \left(\frac{4}{\Delta t^2} + \frac{2\alpha}{\Delta t}\right)\Big/\left(\frac{2\beta}{\Delta t} + 1\right)$$

When $\{\Delta x\}$ has been determined, the increment of nodal displacement is as follows

$$\{\Delta r\} = \frac{1}{\frac{2\beta}{\Delta t} + 1}(\{\Delta x\} + 2\beta\{\dot{r}_0\}) \tag{10}$$

Hence, the increments of velocity and displacement go as

$$\{\Delta\dot{r}\} = \frac{2}{\Delta t}\{\Delta r\} - 2\{\dot{r}_0\}$$

$$\{\Delta\ddot{r}\} = \frac{4}{\Delta t^2}\{\Delta r\} - \frac{4}{\Delta t}\{\dot{r}_0\} - 2\{\ddot{r}_0\}$$

$[M_2]$ and $[K_2]$ need not be formed and saved in memory when the computation is implemented. The element mass and stiffness matrixes are multiplied by element velocity vector of the material with the second damping properties, then the result is added to the right side of Eq. (9). From this point, we can see that the fewer the elements with the second damping property compared with the total elements of the structure, the more efficient the integral algorithm.

For the structures with only one damping property material, Eqs. (7) and (9) become

$$\left[(\frac{4}{\Delta t^2} + \frac{2\alpha}{\Delta t})[M] + (\frac{2\beta}{\Delta t} + 1)[K]\right]\{\Delta r\} = \{\Delta P\} + 2[M]\left[\{\ddot{r}_0\} + (\frac{2}{\Delta t} + \alpha)\{\dot{r}_0\}\right] + 2\beta[K]\{\dot{r}_0\} \tag{7'}$$

$$(\gamma[M] + [K])\{\Delta x\} = \{\Delta P\} + 2[M]\left[\{\ddot{r}_0\} + (\frac{2}{\Delta t} + \alpha - \beta\gamma)\{\dot{r}_0\}\right] \tag{9'}$$

They are the same as the equations used in

DRAIN-2D[7].

The algorithm above has been realized in the computer program DAS (Dynamic Analysis of Structures) devloped by the authors.

3 Example

A three-dimensional structural system of a 10 storey RC frame with a steel tower atop it is considered, as depicted in Fig. 1 and Fig. 2. The total height of the RC building is 37. 82 m; the inter-storey height is 4. 88 m for the first floor and 3. 66 m for the others. The geometric scheme of the frame, the material characteristics and mass values at each floor level are the same as those in Ref. [8] , but unlike Ref. [8] , there is no RC wall in the structure. The steel tower is divided into seven segments. The elevation, stiffness and mass at the top of each segments is listed in Tab.1.

Two different damping cases are examined in this paper. One case is the real damping system (0. 05 damping ratio for RC building and 0. 01 for steel tower) , the other is the equivalent damping system (0.03 for the whole system). The two cases are subjected to the N-S component of the 1940 El-Centro earthquake ground motion scaled to 0. 07 *g*, in *X* direction (Fig. 3).

The example is analyzed using the computer program DAS.

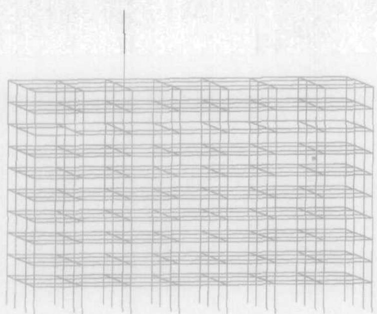

Fig. 1 Structural layout of the 10-storey RC frame with a steel tower atop it

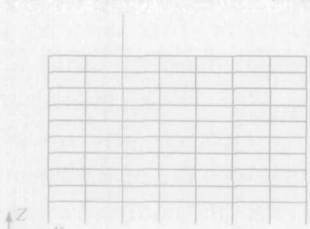

Fig. 2 Elevation of the example

Tab. 1 Property of the steel tower

Segment	$EI/(\mathrm{kN \cdot m^2})$	Elevation/m	Mass/t
7	34 660	70. 82	5
6	245 600	65. 82	10
5	906 600	60. 82	15
4	1 383 000	55. 82	20
3	2 780 000	50. 82	30
2	5 980 000	45. 82	40
1	1. 980 000	42. 82	50
Roof of RC frame		37. 82	

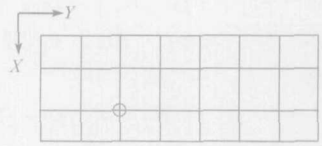

Fig. 3 Roof plan of the example

For the purpose of illustration, the time history of relative horizontal displacement at the top of the steel tower and the roof of the RC frame are shown in Figs. 4—9.

From Fig. 4 and Fig. 5 , we can see that the maximum displacement response at the top of the steel tower calculated with real damping is greater than that calculated with equivalent damping. But the maximum displacement at the roof of the RC frame is just on the contrary. The same result is observed by comparing Fig. 6 with Fig. 7. The relative displacement responses between the top of the steel tower and the roof of the RC frame for the two damping cases are plotted in Fig. 8.

From Fig. 4—Fig. 8 , we can see that the relative displacement responses calculated with equivalent damping are significantly smaller than those obtained with real damping model. This is also true for the relative acceleration responses between the top of the steel tower and the roof of the RC frame shown in Fig. 9.

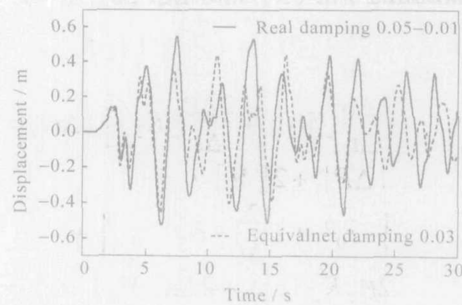

Fig. 4 Horizontal displacement at the top of the steel tower in *X* direction

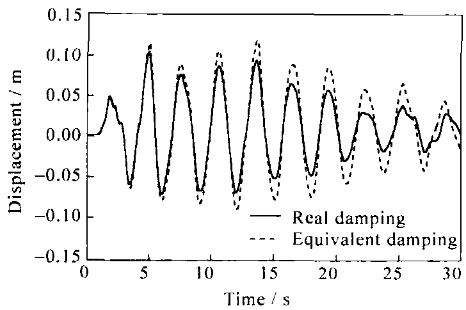

Fig. 5 Horizontal displacement at the roof of the RC frame in *X* direction

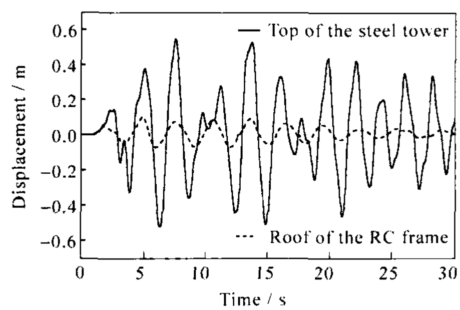

Fig. 6 Horizontal displacements at the top of the steel tower and the roof of the RC frame with real damping in *X* direction

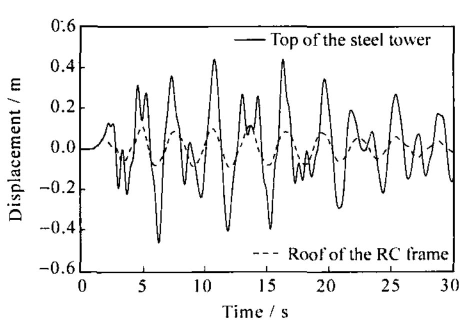

Fig. 7 Horizontal displacements at the top of the steel tower and the roof of the RC frame with equivalent damping in *X* direction

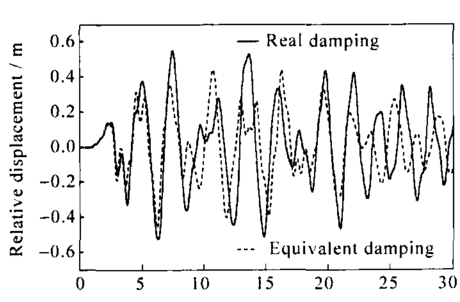

Fig. 8 Relative displacements between the top of the steel tower and the roof of the RC frame in *X* direction

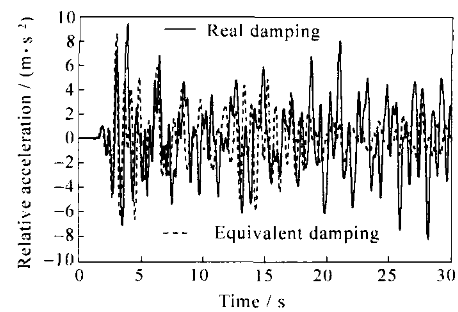

Fig. 9 Relative accelerations between the top of the steel tower and the roof of the RC frame in *X* direction

These results suggest that the equivalent damping model cannot simulate the true situation where the structural parts with less damping ratio have greater response, while the structural parts with greater damping ratio have smaller response.

Tab. 2 shows the maximum displacement responses at the top of the steel tower, the roof of the RC frame, and the difference between them. We can see that the displacement response of the steel tower calculated by using real damping model is greater than that calculated by using equivalent damping model. This indicates that the whiplash effect of non-proportional damping systems is greater than that of equivalent damping systems. It seems that the lower damping ratio of the steel tower is accounted for the greater whiplash effect.

Therefore, if equivalent damping ratio is used in design, the result will be unsafe for the steel tower atop the RC building.

Tab. 2 Maximum displacement response m

Response	Real damping	Equivalent damping 0.03	Equivalent damping 0.05
Top of steel tower	0.546	0.439	0.320
Roof of RC frame	0.103	0.117	0.103
Displacement difference *	0.471	0.357	0.255

Note: * the maximum displacement difference at each time interval.

Fig. 10 and Fig. 11 show the acceleration responses of the structures at the top of the steel tower and the roof of the RC frame. Fig. 10 and Fig. 11 are obtained by using the real damping model and equivalent damping model, respectively. The maximum value is 9. 177 m/s^2 for the real damping model, and 8. 494

m/s^2 for the equivalent damping model. The former is 1.08 times of the later. If the acceleration vector composition is used in *X* and *Y* directions, the value is 1.12. This means that if the equivalent damping model is used in design, the earthquake motion at the steel tower will be less than the real situation by 12%. So the non-proportional damping model should be used for these structures.

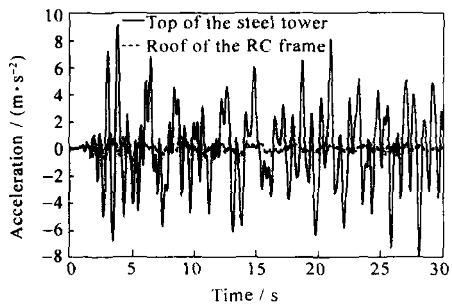

Fig. 10 Acceleration responses at the top of the steel tower and the roof of the RC frame in *X* direction with real damping model

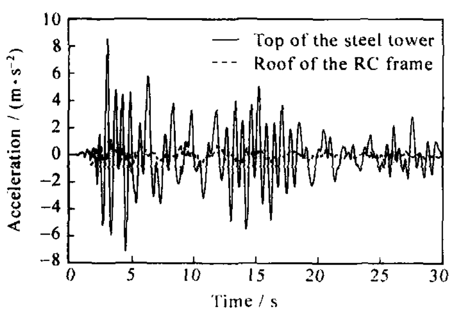

Fig. 11 Acceleration responses at the top of the steel tower and the roof of the RC frame in *X* direction with equivalent damping model

4 Conclusions

Our analysis indicates that the equivalent damping model cannot simulate the structural system of an RC building with a steel tower atop it with acceptable accuracy. Instead,the non-proportional damping model should be used for this structural system. The result by the equivalent damping model would be unsafe for the steel tower and too safe for the RC building at the same time. If there is no suitable software with non-proportional damping model for design, the inner force and displacement of the steel tower atop RC building should be amplified based on the calculated results obtained by using general structural software.

References

[1] Clough R, Penzien J. *Dynamics of Structures* [M]. 2nd ed. McGraw-Hill Inc, 1993.

[2] Wilson E L. *Three Dimensional Static and Dynamic Analysis of Structures* [M]. 3rd ed. CSI Computer and Structures Inc, Berkeley, California, 2002.

[3] Ibrahimbegovic A, Wilson E L. Simple numerical algorithms for the mode superposition analysis of linear structural systems with non-proportional damping[J]. *Computer and Structures,* 1989,33 (2): 523—531.

[4] Claret A M, Venancio-filho F. A modal superposition pseudo-force method for dynamic analysis of structural systems with non-proportional damping [J]. *Earthquake Engineering and Structural Dynamics,* 1991, 20:303—315.

[5] D'Aveni A, Muscolino A. Improved dynamic correction method in seismic analysis of both classically and non-classically damped structures [J]. *Earthquake Engineering and Structural Dynamics,* 2001, 30: 501—517.

[6] Wang Yiqun, Li Zhongxian. Elasto-plastic dynamic analysis of composite structures made up of materials of different damping properties [J]. *Earthquake Engineering and Engineering Vibration,* 1999, 19(2): 76—80(in Chinese).

[7] Kanaan A E, Powell G H. General Purpose Computer Program for Inelastic Dynamic Response of Plane Structures[R]. Report No. EERC73-6, April 1973.

[8] Pinto P E. *Seismic Design of Concrete Structures* [M]. CEB Technical Press, London, 1987. 62—64.